Community Investments and Collusion

Marcus Davidsson

E-mail: davidsson_marcus@hotmail.com

ABSTRACT

This paper will present a simple coin toss model which will illustrate the effect of collusion and its impact on the expected payoff for any given player. Such a game has direct links to community investment clubs where people are aggregating their investments. Without any knowledge about the impact of collusion such behaviour and its significant can be hard to detect. This paper will also review some previous studies on the topic of collusion.

Keywords: coin tossing, collusion, game theory, investment

1. INTRODUCTION AND LITERATURE REVIEW

Collusion is not only a theoretical game theory concept. It can be found in many different industries around the world and has important applications for investors, auctioneers, business owners and policy makers etc. There exist many studies that have investigated collusion strategies and its effects. One way to understand collusion and its effect is to build mathematical models that try to incorporate some of the characteristics found in oligopoly markets. Shone (2003) describes a duopoly Cournot model that incorporates collusion and shirking decisions. Both firms in the model use their profit function to derive reaction functions. The interaction point of these reaction functions is called the Cournot/Nash equilibrium. Such point represents the profit maximization without any collusion. By applying collusion strategies, i.e. creating a price cartel, the two firms can increase their profits even more on the behalf of the consumer. The duopoly will then effectively become a monopoly. Such price competition is in many countries illegal. Roos (2004) looks at heterogeneous firms in a dynamic model of collusion. The firms can make market entry and exit decisions, investment decisions and engage in collusion based upon their own preferences. The author finds that that the firms will wait until they have a market share on par with its competitors before collusion agreements are made. Collusion in general tends to hurt the consumers by driving out competitors giving the consumer higher prices and excess revenue to the colluding firms.

Fershtman and Pakes (2000) further explain that society tends to be better off when firms do not collude due to lower prices induced by perfect competition. However, for small markets that only support a small number of businesses collusion in some cases can be preferable from a social welfare perspective in order to create high-quality products. There also exist studies that have analyzed collusion from an empirical perspective. Chen and Ritter (2000) look at data for Initial Public Offering (IPO) in the United States and in the world. They found that the gross spreads received by underwriters on IPO’s in the United States are much higher than in other countries. They also found that 90% of the IPO’s has gross spreads of exactly seven percent. They interpret this as significant collusion is taking place between the US investment banks since they don’t want to compete on fees. Hansen (2001) however does not support the collusion hypothesis in IPO’s. Dutta and Madhavan (1997) analyze stockprices and market makers inter-temporal pricing strategies. They found that market makers that engage in non-cooperative pricing strategies will set the bid-ask spread above competitive levels. They define this action as implicit collusion. This is different from explicit collusion where the market makers agree on fixed pricing rules. The authors also explain that asymmetric information is not required for collusion to occur. Eckbo (1983) tests the hypothesis that horizontal mergers, which is the merger between companies producing similar goods, results in a higher probability of successful collusion among rival producers when the number of competitors are reduced. The author uses a large dataset containing horizontal mergers in the mining and manufacturing industries. He find little evidence of a relationship between horizontal mergers and increased collusion. Slade (1992) investigates pricing strategies in the Vancouver retail-gasoline market. The author finds that during normal market conditions
prices trend to be comparatively stable i.e. collusion but during periods of demand/supply shocks price wars tend to break out. Other authors such as Gasmi, Laffont and Vuong (1992) have studied collusive behaviour in the soft-drink market. They used data for Coca-Cola and Pepsi-Cola from 1968 to 1986. They found that a collusion hypothesis could not be rejected. Bresnahan (1987) investigates the presence of collusion in the American automobile industry. The author found that the collusion hypothesis is confirmed for the years 1954 and 1956 but the same hypothesis is rejected in 1955 due to a sudden supply-side shock. Baker (1989) finds evidence of cartel pricing in the US Steel Industry in 1935 to 1939.

Collusions also play a large role in auction theory. Robinson (1985) looks at collusion among bidders in auctions. In a sealed-bid first price auctions the potential buyers submit closed offers and the product is sold to the highest bidder. In a sealed-bid second price auction the highest bidder only pays what the second-highest bidder offered. The author explains that collusion among bidders can be minimized if the seller uses sealed-bid second price auctions rather than sealed-bid first price auctions. Marshalla and Marx (2007) also discuss collusion in first-price and second-price auctions. The author shows that all ring competition at second-price auctions can be eliminated if the cartel cannot control the bids of their members. This is however not true for first-price auctions. Lind and Plott (1991) describes a phenomenon call the "winner's curse". In a common-value auction the value of the item is unknown. The “winner's curse” arises when the winners of the auction has placed a bid that is higher than the value of the item. The authors argue that such a phenomenon can be seen in the auctions for natural resources i.e. mineral rights. The value of the mineral is unknown so each firm has an estimate it.

Hansen (1986) uses data from the U.S. Forest Service timber auctions to test the hypothesis that sealed-bid and open auctions yield equal revenue. The author uses ordinary least squares, two stage least squares, and maximum likelihood to show a statistically insignificant difference in high-bids between sealed-bid and open auctions. Klemperer (2002) notes that collusion between bidders can be minimized by making sure that the auction design is well thought out. Auction design is also important in order not to exclude potential bidders from participating in the auction. The author takes the European 3G mobile-phone licence auctions as an example of poor auction design since there where very large differences in revenue generated by the European governments even though the value of this licences where similar. Poor auction designs in some countries resulted in increased collusion between participating firms which lead to the government revenues i.e. taxpayer money where reduced.

2. COLLUSION MODEL
In order to illustrate the effects of collusion we will now consider a simple coin toss game (Leighton & Lehman, 2004). Three people are playing a coin toss game. Let’s call them Dan, Eric and Nick. The objective is to bet on the outcome of the coin toss. Each person write down on a piece of paper what they think the outcome of the coin toss is going to be (head or tail) Each person has to pay 2 $ to participate in the game and if someone correctly predicts the outcome of the coin toss he will win the pot ( 2+2+2= 6$ ). If two people correctly predicts the outcome each person gets 3 $ (6$/2) and if all three people correctly predicts the outcome each person gets 2 $ (6$ / 3) which is initial cost of the game. We are going to start analyzing a fair game where no collusion is taking place. In exhibit-1 we can see Dan's payoff given all the different outcomes of Dan's, Eric's and Nick's predictions. Note that we denote Yes=Y and No=N. Note also that we define payout as the amount of money each person gets if they make the correct prediction minus the initial cost of the game. For example: If all the guys make the correct prediction Dan's payout is 2$ -2$ =0. If Dan makes the right prediction, Eric makes the right prediction and Nick fails then Dan's payout is 3 $ - 2 $ = 1 $. If Dan makes the right prediction, Eric fails and Nick fails then Dan's payout is 6 $ - 2 $ = 4 $. Note that if Dan's prediction fails he can only lose the initial stake which is 2 $. Note also that all of these different outcomes have an equal chance of happening so the probability of each outcome is 1/8. Since the game is completely random Dan's expected payoff \( E(Pay_D) \) should be zero. So over time Dan should not win or lose any money.

\[
E(Pay_D) = (0) \times (1/8) + (1) \times (1/8) + (1) \times (1/8) + (4) \times (1/8) + (-2) \times (1/8) + (-2) \times (1/8) + (-2) \times (1/8) + (0) \times (1/8) = 0
\]
EXHIBIT-1: Dan’s Payout without Collusion (Leighton & Lehman, 2004)

<table>
<thead>
<tr>
<th>Eric right?</th>
<th>Nick right?</th>
<th>Dan’s payoff</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1/2</td>
<td>$0</td>
<td>1/8</td>
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<tr>
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<td>N</td>
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<td>N</td>
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<tr>
<td>N</td>
<td>Y</td>
<td>$0</td>
<td>1/8</td>
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</tbody>
</table>

EXHIBIT-2: Dan’s Payout with Collusion (Leighton & Lehman, 2004)

<table>
<thead>
<tr>
<th>Eric right?</th>
<th>Nick right?</th>
<th>Dan’s payoff</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1/2</td>
<td>$0</td>
<td>0</td>
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<tr>
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<td>1/4</td>
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<tr>
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<td>$4</td>
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<td>-$2</td>
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<td>Y</td>
<td>$0</td>
<td>0</td>
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</tbody>
</table>
EXHIBIT-3: Payouts defined in a Programmatic Fashion

We can now introduce collusion. Eric and Nick are collaborating with each other. Nick always takes the opposite bet of Eric. So for example if Nick bets Head Eric will bet Tail. This collusion behaviour means that the assumption of independency is broken. This will have 2 different consequences.

i. Both Eric and Nick’s predictions cannot be correct in the same game since they make opposite predictions.

ii. Both Eric and Nick’s predictions cannot be wrong in the same game since they make opposite predictions.
This means that Dan will never alone win a pot. He will always share the pot with someone else i.e. Eric or Nick. If we correct Exhibit-1 for these two identified consequences we will get the tree diagram in Exhibit-2 which illustrates Dan’s payout. We can see that only 4 different outcomes are valid. This means that the probability of these outcomes is reduced to 1/4. We can now calculate Dan's expected payoff when he is exposed to collusion

\[ E(Pay_D) = (0) \times (0) + (1) \times (1/4) + (1) \times (1/4) + (4) \times (0) + (-2) \times (0) + (-2) \times (1/4) + (-2) \times (1/4) + (0) \times (0) = -0.5 \]

This means that Dan will on average lose 0.5$ per game which means that Eric and Nick expected value must be +0.5. Eric and Nick will on average win 0.5$ per game even if the coin toss is completely random. A 0.5$ average winning of Eric and Nick is equal to an average winning per person of 0.25 $. We have therefore shown that a collusion strategy can turn a zero expectancy game into a positive expectancy game. Now in order to be able to simulate such a game we need to express the payouts in a more quantitative form as seen in Exhibit-3. We can now run a simulation which contains such payouts as seen in Exhibit-4. We start by simulating the case when we have no collusion and a game cost of 2 $. The expected values for the three players are close to zero as seen in the blue plots. We now assume that player 2 and 3 are colluding but they do not share their profit / losses. Player 1’s expected payout becomes -0.5 and player 2 and player 3’s expected payout becomes +0.25 as seen in the red plots. We now assume that player 2 and 3 are colluding and they share their profit / losses. The expected payout are the same as in the previous example even though the payouts are different as seen in the green plots.

**EXHIBIT-4:** Simulation Output from Collusion Coin Toss Game
3. CONCLUSIONS AND FINAL DISCUSSION

We have in this paper discussed collusion with the help of a simple coin tossing game which has direct applications to community investment clubs where people aggregate investments and share outcomes. We have seen that collusion shift the expected value from zero to positive for colluding players. In real life an uninformed player might have problems detecting collusion even though collusion will have a significant impact on his expected value. Another problem with dealing with such a simple game as in this paper is that we assume that player-1 selection is completely random. However, there does not exist anything that guarantee that player-1 selection is completely random. If player-2 and player-3 systematically takes the opposite bet, player-1 might become suspicious and also start to make opposite selections in regards to one of the other players. Such behaviour will lead to that the game breaks down. When we have more players such a collusive behaviour might become even more obscure which makes it difficult to detect if you are not specifically looking for it.

We can also see in appendix-1 how the expected value of two colluding players changes when the cost of the coin toss game increases. We can see that the more costly it is to participate in the game the higher the expected value of the two colluding players becomes. In appendix-2 we can see the dynamics for a 1-2 coin toss game where player two always selects 2 and where player three always selects 1. The probability that player one will select 1 depend on a Bernoulli coin toss with probability p. We can see the expected value for the three players given different game costs and different probabilities that player one’s selection will be the same as player two’s selection. Player three’s expected value increases when the probability that player one’s selection will be the same as player two’s selection. The reason for that is because player three will win fifty percent of the time and when player three wins he will always win by himself. When player one and player two wins they win together which means that their payoffs will be lower. The only two alternative are [player one right, player two right, player three right]; [yes, yes, no] and [no, no, yes].

REFERENCES

Shone, R (2003) Economic Dynamics - Phase Diagrams and their Economic Applications
Appendix-1

![Graph of Cost Coin Toss Game for Player 1]

![Graph of Expected Value Player 2 & 3]

Appendix-2

![3D Graph of Cost Game]

![Graph of Expected Value Player 1 vs. Prob P1=P2]