Mean-Variance and Mean-Gini Analyses to Portfolio Optimization in Malaysian Stock Market

DR. SAIFUL HAFIZAH JAAMAN & WENG HOE LAM
School of Mathematical Sciences, Faculty of Science and Technology,
Universiti Kebangsaan Malaysia, 43600 UKM Bangi,
Selangor, Malaysia.
E-mail: shj@ukm.my

Acknowledgement:
This study is supported by Universiti Kebangsaan Malaysia’s Research Grant Code UKM-GUP-2011-223

ABSTRACT

The mean-variance (MV) model is commonly used in portfolio optimization for comparing uncertain prospects. This model however relies strictly on the assumptions that the returns of assets follow normal distribution or the investor’s utility function is quadratic. In reality these two conditions do not hold. The mean-Gini (MG) model has been proposed to overcome the limitations of the mean-variance model. In this paper, portfolio compositions and performances employing the MV and MG models utilizing data from the Malaysian share market are compared. Results of this study show that the MG portfolio outperforms the MV portfolio. MG model is not restricted to normal distributions and quadratic utility function enabling investors to construct second degree stochastic dominance (SSD) efficient portfolios thus MG model is a better alternative model for risk-averse investors.

Keywords: investment, return, risk, portfolio composition, portfolio performance

1. INTRODUCTION

Investors like to focus on the promise of high returns, but they should also ask how much risk they must assume in exchange for these returns. Since its introduction by Markowitz (1952) the mean-variance analysis (MV) has served as the standard procedure for constructing portfolios. Due to its simplicity the MV model still dominates and has a prominent place in finance to this date. The MV analysis requires investors to estimate the means and variances of all assets and the covariances of all assets pairs. Markowitz demonstrated that in the dimensions of expected return and standard deviation, efficient portfolios that maximize expected returns can be determined for given levels of risk. If the portfolio returns are normally distributed or investors have quadratic utility, the MV approach to portfolio construction is then acceptable for maximising expected utility. The MV model relies strictly on the assumptions that assets returns are normally distributed or the investor’s utility function is quadratic. If these two conditions do not hold, then MV model will not be consistent with the principle of expected utility maximization (Tobin, 1958). However, studies such as by Arditti (1971), Simkowitz and Beedles (1978), Chunhachinda et al. (1997), and Prakash et al. (2003) have shown that assets returns are not normally distributed. Study done by Jaaman et al. (2011a, 2011b) on the Malaysian market found that skewness to be an important factor. Moreover, quadratic utility is not a realistic description of investor’s attitude toward risk. It assumes investors are as averse to upside deviations as they are to downside deviations. Hanoch and Levy (1969) reported that the quadratic utility function implied that investors prefer less wealth to more wealth. In addition, quadratic utility assumes investors have increasing absolute risk aversion.

The mean-Gini (MG) of portfolio analysis was proposed by Shalit and Yitzhaki (1984) to overcome the shortcomings of MV model. The MG model is not restricted to normal distributions or quadratic utility function. Hence, MG is potentially a better framework than MV. The MG framework is similar to MV in that it depends on two statistics; the mean and a measure of dispersion, to characterise the risky assets. With Gini as the measure of risk to be minimized, subject to a given mean return, the MG provides sufficient conditions for second degree stochastic dominance (SSD) thus, investors can rank portfolios consistently to individual preferences and be prevented from choosing inferior portfolios (Dorman 1979 & Cheung et al. 1990). Studies done on MG model include by Yitzhaki (1983), Shalit & Yitzhaki (1986), Shalit & Yitzhaki (1989), Okunev (1991), Ringuest et al. (2004) and Shalit & Yitzhaki (2005).
The objective of this paper is to compare the portfolio compositions and performances constructed employing the mean-variance (MV) and the mean-Gini (MG) models using data from an emerging market, the Malaysia stock market. The remaining of the paper is organized as follows. In the next section, the concepts and mathematical models of MV and MG are discussed. Section 3 describes the data and methodology used in this study. Section 4 presents the empirical results and finally section 5 concludes the paper.

2. RISK MEASURES

2.1 Variance

Markowitz (1952) introduced the MV model by using variance as the risk measure. The mean and variance are the two important summary statistics of MV model where the mean represents the reward while variance represents the variability. Employing these two statistics, it is simple to generate the efficient frontier. The MV model is a quadratic programming model. The objective function of MV model is to minimize the portfolio variance. The covariance matrix of assets returns need to be calculated in order to compute the portfolio variance. The mathematical model is as follows:

\[
\text{minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \\
\text{subject to } \sum_{j=1}^{n} r_j x_j \geq \rho , \\
\sum_{j=1}^{n} x_j = 1, \\
x_j \geq 0.
\]

where \( \sigma_{ij} \) is the covariance between assets \( i \) and \( j \), \( x_j \) is the amount invested in asset \( j \), \( r_j \) is the expected return of asset \( j \) per period, \( \rho \) is a parameter representing the minimal rate of return required by an investor.

2.2 Gini’s Mean Difference

Gini’s mean difference is a statistic that is widely used in measuring income inequality. Yitzhaki (1982) proposed the MG model using Gini’s mean difference as risk measure in portfolio optimization. Gini’s mean difference is the average of the absolute differences between all possible pairs of observations of a random variable. Gini’s mean difference is defined as follows:

\[
\tau = \frac{1}{2} \int_{a}^{b} \left| X - Y \right| f(X)f(Y) dX dY
\]

or in discrete case

\[
\tau = \frac{1}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left| Y_j - Y_k \right|
\]

where \( f(X) \) and \( f(Y) \) are probability density functions of \( X \) and \( Y \).

The MG efficiency criteria are that distribution \( F \) dominates distribution \( G \) if \( \mu_F \geq \mu_G \) and \( \mu_F - \tau_F \geq \mu_G - \tau_G \), where \( \mu \) is the mean and \( \tau \) is half Gini’s mean difference. These two criteria are necessary conditions for first and second degree stochastic dominance. The MG model has advantages over MV model. Yitzhaki (1982) has showed that the MG efficient set is consistent with the second degree stochastic dominance (SSD) rule. The MG efficient set is the subset of SSD efficient set. SSD efficient set is ideal for risk-averse investors because there are no restrictions on the assets returns distribution and investors exhibit risk-averse utility function (Hadar & Russell 1969). The MG approach is as simple as MV because it also uses two summary statistics which are mean and Gini’s mean difference. Furthermore, the MG approach can be used to derive capital asset pricing models (Shalit & Yitzhaki 1984). The objective of MG model is to minimize the Gini’s mean difference. The MG mathematical model is presented as follows:

Minimize

\[
\frac{1}{N^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left| Y_j - Y_k \right|
\]
subject to \( \sum_{i=1}^{I} \alpha_i \mu_i = \mu, \)
\( \sum_{i=1}^{I} \alpha_i = 1, \)
\( \alpha_i \geq 0. \) \hspace{1cm} (4)

Using Hazell (1971) approach, model (4) is equivalent to the linear programming model (5) by removing the absolute value term from the objective function.

\[
\text{minimize} \quad \frac{1}{N} \sum_{j=1}^{N} \sum_{k=1}^{N} (\alpha_{jk}^+ - \alpha_{jk}^-) \\
\text{subject to} \quad \sum_{i=1}^{I} \alpha_i (X_{ij} - X_{ik}) = \alpha_{jk}^+ + \alpha_{jk}^- \\
\sum_{i=1}^{I} \alpha_i \mu_i = \mu, \\
\sum_{i=1}^{I} \alpha_i = 1, \\
\alpha_i, \alpha_{jk}^+, \alpha_{jk}^- \geq 0. \] \hspace{1cm} (5)

where \( N \) is the number of periods, \( Y_j \) is the portfolio return during period \( j \), \( X_{ij} \) is the return of asset \( i \) during period \( j \), \( \mu_i \) is the mean return of asset \( i \), and \( \mu \) is the portfolio mean return.

3. DATA AND METHODOLOGY
The data for this study is drawn from Bursa Malaysia, consists of the monthly returns from July 2002 to December 2007 of twenty-nine shares included in the Kuala Lumpur Composite Index (KLCI). KLCI is the main index for Malaysian market acting as a barometer that measure the performance of the major capital and industry segments of the Malaysian and regional markets. Furthermore, the up and down movements of KLCI reflect how investors feel about the economy. The MV and MG optimal portfolios are constructed using models (1) and (5). The portfolio performance is calculated using reward per risk equation as follows:

\[ \text{Portfolio performance} = \frac{\text{mean return}}{\text{risk}} \] \hspace{1cm} (6)

Lingo software is utilized to obtain the optimal portfolio compositions for the two models. The minimum required rate of return is set as 0.02, higher than the KLCI average return during this study period.

4. EMPIRICAL RESULTS
4.1 Portfolio performances
Table 1 presents the summary statistics of the optimal portfolios constructed by the MV and MG risk measures. Both MV and MG portfolios achieve the target mean return of 0.02 set in this study. The MV portfolio is deemed to be more risky (risk of 0.0359) than the MG portfolio (risk of 0.0201). This result is in accordance to studies done by Shalit and Yitzhaki (1989), Shalit and Yitzhaki (2005) which report that MG portfolio gives lower risk. MG portfolio is SSD efficient and consistent with the maximization of expected utility principle as reflected by the higher performance of MG portfolio (0.9970) compared to the MV portfolio (0.5565).

4.2 Portfolio compositions
An investor can reduce risk of his investment by spreading it over a number of securities. The question arises then that if investor is able to form a diversified portfolio and not constrained to a single security, how then will he allocate his fund among the various alternatives. In this study MV and MG models are employed to construct optimal portfolios from 29 available firms. Table 2 shows the optimal portfolio compositions of both MV and MG models.

Both MV and MG models present different optimal portfolio compositions. Out of 29 shares, 8 shares are chosen in the optimal portfolio constructed using MV model while 7 are selected in the optimal portfolio.
generated by MG model. According to Byrne and Lee (2004) the different portfolio compositions are due to the non-normality of the data. In both MV and MG portfolios SHELL is the most dominant share with 24.09% of fund invested for MV portfolio and 23.82% for MG portfolio. KLK share (1.23%) is the smallest component in MV portfolio while no fund is allocated in KLK in MG portfolio. As for MG portfolio KULIM, with 7.02% investment, is the least dominant share.

5. CONCLUSION
This paper discusses and compares the optimal portfolio compositions as well as performances employing MV and MG models for the Malaysian market. As discussed, both models give different portfolio compositions. The results show that the MG portfolio outperforms the MV portfolio. MG model is shown to have better theoretical properties and practical applications than MV model. Moreover, MG model is not restricted to normal distributions and quadratic utility function enabling investors to construct SSD efficient portfolios by using the MG approach thus MG model is a better alternative model for risk-averse investors.

Table 1: Summary Statistics of Optimal Portfolios

<table>
<thead>
<tr>
<th></th>
<th>MV</th>
<th>MG</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean return</td>
<td>0.0200</td>
<td>0.0200</td>
</tr>
<tr>
<td>Risk</td>
<td>0.0359</td>
<td>0.0201</td>
</tr>
<tr>
<td>Performance</td>
<td>0.5565</td>
<td>0.9970</td>
</tr>
</tbody>
</table>

Table 2: Percentage of Stocks in Optimal Portfolios

<table>
<thead>
<tr>
<th></th>
<th>MV</th>
<th>MG</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAT</td>
<td>7.27</td>
<td>8.74</td>
</tr>
<tr>
<td>IOICORP</td>
<td>13.88</td>
<td>15.06</td>
</tr>
<tr>
<td>K.L.K</td>
<td>1.23</td>
<td>0</td>
</tr>
<tr>
<td>KULIM</td>
<td>4.44</td>
<td>7.02</td>
</tr>
<tr>
<td>MISC</td>
<td>18.08</td>
<td>18.37</td>
</tr>
<tr>
<td>SHELL</td>
<td>24.09</td>
<td>23.82</td>
</tr>
<tr>
<td>SIME</td>
<td>17.23</td>
<td>13.59</td>
</tr>
<tr>
<td>UMW</td>
<td>13.78</td>
<td>13.40</td>
</tr>
</tbody>
</table>

REFERENCES


